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Formation of Multiple Populations in Globular Clusters: constraints on the dilution by pristine gas

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ABSTRACT

The star-to-star differences in the abundance of light elements observed in the globular clusters (GCs) can be explained assuming that a second generation (SG) of stars form in the gas ejected by the asymptotic giant branch (AGB) stars belonging to a first stellar generation. However, while Na and O appear to be anticorrelated in the cluster stars, from the stellar models they turn out to be correlated into the AGB ejecta. In order to reconcile the stellar theory with the observational findings, all the GC models invoke an early dilution of AGB ejecta with pristine gas occurring during the SG formation. Despite a vast consensus about the occurrence of such dilution, the physical process behind it is still unknown. In the present paper we set some general constraints on the pristine gas dynamics and on the possible amount of pristine gas involved in the SG formation, making use of a one zone chemical model. We find that such a dilution is a necessary ingredient in the SG star formation to explain the observed abundance patterns. We confirm the conclusion of our previous works showing that clusters must have been initially much more massive. We also show that models assuming that clusters had an initial mass similar to their current one, and adopting a large fraction of pristine gas to form SG stars, fail to reproduce the observed Na-O anticorrelation and are not viable. We finally show that the dilution event should be restricted in time, rather than extended for the all duration of the SG formation.

Key words: stars: chemically peculiar – globular clusters: general

1 INTRODUCTION

In the last decade a number of spectroscopic data provided the evidence of spreads of light elements and anticorrelations between Na and O and Mg and Al in the stars of globular clusters (GCs), indicating the presence of at least two stellar populations in these objects. As these properties are not shared by the halo stars with the same metallicity, nor by open clusters in our or other galaxies, they represent a trait of the GCs so peculiar to lead Carretta et al. (2010) to identify GCs as those clusters where there is a Na-O anticorrelation. On the basis of this anticorrelation, Carretta et al. (2010) separate the GC stars in three families: a Primordial population of stars sharing the same abundances of the halo stars; an Intermediate population of stars with a lower O abundance and higher Na; and an Extreme population of very O-poor stars ([O/Fe]< -0.4).

The variations are also found among unevolved stars currently on the main sequence (MS). This implies that the above chemical characteristics have been imprinted in the gas by a previous generation of stars, because low-mass MS stars do not reach the high temperatures needed to sustain the nuclear reactions leading to the observed relations between the elements.

It is still debated which kind of stars act as polluters, whether asymptotic giant branch (AGB) stars (D'Antona & Ventura 2007) or fast rotating massive stars (FRMS; Decressin et al. 2007b). In D'Ercole et al. (2008), we presented a model for the formation and dynamical evolution of multiple populations in GCs in which second generation (SG) stars form out of the AGB ejecta of the first generation (FG) stars. We first focussed on the hydrodynamic of the ejecta that collect into the cluster core through a cooling flow; it turned out that, in order to shed enough gas to form the observed SG, the FG should have been originally about ten times larger than that observed today. By means of N-body simulations we then showed how the clus-

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ter can lose FG stars reaching the nowadays observed ratio of the number of FG and SG stars.

In a successive paper (D'Ercole et al. 2010, hereafter Paper I), we focused on the chemical evolution of the GCs through simple one-zone models, and we were able to reproduce the main characteristics of clusters such as NGC 2808 and M4. The main problem encountered in this kind of models is given by the fact that the current stellar models indicate that during the AGB phase [O/Fe] and [Na/Fe] in the ejecta are always correlated (e.g. Karakas & Lattanzio 2007; Ventura & D'Antona 2009; Siess 2010; Ventura & D'Antona 2011). Thus, the O and Na abundances in the cluster interstellar medium (ISM) are correlated, and the SG stars forming in this gas would exhibit the same correlation, in glaring conflict with the observations. Also in the FRMS scenario some degree of dilution with pristine gas is needed to account for the presence of lithium in SG stars because this fragile element is absent in the massive star ejecta (Decressin et al. 2007a; Lind et al. 2011). To date, dilution seems to be required by all the proposed models.

Dilution was actually implemented in the models presented in Paper I. This implementation, however, was realized via a reasonable but arbitrary modelling. In fact, despite the general consensus about the need of dilution, the source and dynamics of the diluting pristine gas is still not clear, and the mechanisms proposed till now (D'Ercole et al. 2008; Bekki & Mackey 2009; Pflamm-Altenburg & Kroupa 2009; Conroy & Spergel 2010; Gratton & Carretta 2010) all suffer from some drawbacks.

The aim of the present paper is to shed some light on the dilution process through possible constraints that can be inferred by the chemical properties of the GCs.

2 ON THE NEED OF DILUTION

Given the vagueness of the dilution process and the great uncertainties plaguing some relevant aspects of the nuclear physics during the AGB stage (Ventura & D'Antona 2005a,b, Paper I),one could be tempted to make the simple (but audacious) statement that the actual stellar models are incorrect, and that the Na abundance in the AGB atmosphere decreases in pace with the decrease of stellar mass. In the end, the O-Na anticorrelation shown by the SG stars would be the direct consequence of the O-Na anticorrelation already present in the gas in which these stars form.

Unfortunately, although the above picture eliminates some difficulties connected with the dilution, other problems remain unsolved. While the O-Na anticorrelation seems to be a distinctive characteristic of GCs, the extension of this anticorrelation varies from cluster to cluster (Carretta et al. 2009). In the previous scenario the variation of such extension – that is, of the minimum value $[\mathrm{O/Fe}]_{\mathrm{min}}$ [1] – depends on the time at which the star formation of the SG starts. For the sake of example, we can examine a "fake" evolution of [Na/Fe] within the gas shed by the AGB stars, with the Na abundance decreasing in time and the oxygen which instead grows. Let us now consider two clusters, GC#1 and GC#2, starting to form SG stars at the times t_1 and $t_2 > t_1$, respectively. Clearly, GC#1 will host some very O-poor stars which instead will be absent in GC#2, and therefore the anticorrelation in this latter cluster will be shorter. On the other hand, given our assumption about the Na evolution, the maximum value $[\mathrm{Na/Fe}]_{\mathrm{max}}$ will be larger in GC#1 than in GC#2. Thus, an anticorrelation is expected between $[\mathrm{O/Fe}]_{\mathrm{min}}$ and$[\mathrm{Na/Fe}]_{\mathrm{max}}$, at odd with the observations showing instead a correlation (see Fig. 19 in Carretta et al. 2009).

While this correlation is not quantitatively accounted for by any model yet, we think that in any case it falsifies the starting hypothesis that the anticorrelation O–Na is imprinted in the stellar models. Therefore, despite the problems outlined above, we more safely acknowledge the current models of stellar evolution, and believe that the dilution is an inescapable ingredient of the GC evolution.

3 CONSTRAINTS ON THE FG MASS

As discussed in D'Ercole et al. (2008), in order to explain the current mass of SG stars, globular clusters must have been initially about ten times larger than today[2]. In Vesperini et al. (2010), we have further explored the actual amount of SG stars that a cluster could have formed depending on its initial mass and structural parameters (along with the implications for the contribution of globular cluster FG and SG stars to the assembly of the Galactic halo).

However, Decressin et al. (2007a) leave open the possibility, in the FRMS scenario, of an initial FG mass close to the present one, provided a rather flat IMF. The same hypothesis was put forward by D'Antona & Caloi (2004) assuming the AGB stars as polluters. As shown by D'Ercole et al. (2008), a flatter IMF increases the amount of available ejecta to form the SG population at the cost of reducing the long-lived FG stars, and a large M_{FG} is still needed to raise their number; as a consequence, a flat IMF is not a viable hypothesis to avoid larger clusters in the past.

More recently also Conroy & Spergel (2010) have proposed a model, still based on the AGB scenario, in which the cluster has an initial mass similar to the current one; the SG stars form from the small amount of AGB ejecta available and a very large amount of pristine gas accreted by the GC while moving through the ambient ISM . This last scenario can be easily ruled out for those few clusters in which a very high helium enrichment ($Y \sim 0.38$) has been safely derived from their CMDs because such a large amount of helium can not be released by a number of FG stars as small as the present one (Renzini 2008).

Here we show that, apart from the helium abundance, the assumption of a FG comparable in mass to the present one looks impractical on the basis of the observed anticorrelation between O and Na. Let us assume, as in Paper I, a SG star formation starting at $t = 32$ Myr and lasting up to a fiducial time $t_{\mathrm{end}} = 100$ Myr. During this lapse of time the FG of mass M_{FG} with a Kroupa IMF (Kroupa et al. 1993) sheds an amount of ejecta $M_{\mathrm{ej}} = 0.05 M_{\mathrm{FG}}$ (see

[1] The extension of the anticorrelation is regulated by the value of $[\mathrm{O/Fe}]_{\mathrm{min}}$ as the maximum value is essentially constant in all the clusters and is the same found in the halo stars.

[2] The actual factor depends on the stellar IMF, the duration of the SG formation event, the amount of ejecta a cluster is able to retain (see Vesperini et al. 2010)

D'Ercole et al. 2008). If we now further assume to form an amount of SG stars comparable to the FG, as suggested by Conroy & Spergel (2010), the quantity of the accreted pristine gas must be of the order of $M_{\rm pr} \sim M_{\rm FG}$. In conclusion, the degree of pollution of the SG is about $M_{\rm ej}/M_{\rm pr} = 0.05$, which is definitively too low to explain the observed extension of the O-Na anticorrelation [3].

The above conclusion is highlighted by our chemical one-zone model shown in in Fig. 1 . Adopting the formalism of Paper I, the model is characterized by the following set of parameters:$(t_{\rm end,7}, \rho_{*,\rm FG}, f_{\rm pr}, \nu, x) = (10, 700, 1, 1, 0.5)$. Here $t_{\rm end,7}$ represents the time at which the evolution stops in units of 10^7 yr, $\rho_{*,\rm FG}$ is the FG density in $\rm M_\odot\ pc^{-3}$, $f_{\rm pr}$ is the amount of accreted gas in unit of $\rho_{*,\rm FG}$, ν is the star formation efficiency, and x is the ratio between SG and FG stars. Contrary to the models shown in Paper I, where the accretion rate of the pristine gas is assumed to have a gaussian temporal profile, here we set a constant accretion rate, as expected in the case envisaged by Conroy & Spergel (2010) of a GC moving with constant velocity through an uniform medium.

The top panels of figure 1 show that the FG and SG stars occupy the same region in the [Na/Fe]-[O/Fe] diagram and share the same helium distribution extremely peaked around $Y = 0.246$, the value assumed for the pristine gas and the FG stars. The two populations are thus undistinguishable.

Inhomogeneous mixing has been suggested to overcome the above shortcoming and to obtain the observed anticorrelations (Conroy & Spergel 2010). However, a simple argument shows that resorting to inhomogeneous mixing does not solve this problem. Let us consider an amount of AGB ejecta $M_{\rm ej}$ with a typical sodium abundance [Na/Fe]=0.8 merging with a mass $M_{\rm pr}$ of pristine gas with, for example, [Na/Fe]=0. In case of inhomogeneous mixing, different regions of the mixture would have different values of [Na/Fe], and the stars forming in this gas would have a distribution function N([Na/Fe]) extending across the range 0 <[Na/Fe]< 0.8. Although the shape of this function depends on a number of assumptions (see the Appendix), some conclusions can be drawn independently of its exact knowledge. Carretta et al. (2010) separate the FG from the SG on the basis of their sodium abundance, with the SG stars having [Na/Fe]> 0.4[4]. Thus, assuming that all the mass $M_{\rm ej}$ of the AGB ejecta forms a number $N_{\rm tot}$ of stars, only a fraction $\zeta = N([\mathrm{Na/Fe}] > 0.4)/\mathrm{N_{tot}}$ of them will appear as SG stars, the rest remaining confused with the genuine FG stars. The apparent ratio between first and second generation therefore will be:

$$\frac{M_{\rm SG}}{M_{\rm FG}} = \frac{M_{\rm SG,[Na/Fe]>0.4}}{M_{\rm FG} + M_{\rm SG,[Na/Fe]<0.4}} = \frac{\zeta\xi}{1+(1-\zeta)\xi}. \qquad (1)$$

In the above equation we have assumed that the mass lost by the FG stars (before $t = 100$ Myr) is$M_{\rm ej} = \xi M_{\rm FG}$. From the observations we know that $M_{\rm SG}/M_{\rm FG} \gtrsim 1$, and therefore equation 1 gives rise to the condition

$$0.5\frac{1+\xi}{\xi} \lesssim \zeta \lesssim 1. \qquad (2)$$

This inequality is fulfilled only for $\xi \gtrsim 1$, which is clearly meaningless [5].

In summary, if the initial FG mass was similar to the present one, the observed O-Na anticorrelation cannot be reproduced in any way by inhomogeneous mixing.

To give a "visual" support to the above conclusion, we have implemented an algorithm in our one-zone code in order to take into account inhomogeneous mixing (see the Appendix). The results are shown in the lower panels of Fig. 1. Non homogeneous mixing clearly acts in the right direction, but the number of Intermediate stars turns out to be rather small because of the large overabundance of pristine gas compared to the AGB ejecta[6]. This is particularly appreciable in the helium distribution, which acquire only a small tail toward higher values of Y, although the ejecta of massive AGB stars have values as high as $Y = 0.38$.

We thus conclude that a large FG mass must be a prerequisite of every model of GC evolution, which then must account for how to get rid of the stellar surplus (see D'Ercole et al. 2008).

At the end of this section, we mention the effect of a possible inhomogeneous mixing in models similar to those of Paper I, with an initial $M_{\rm FG}$ much larger than today, and $x \sim 1$. In this case it turns out that, in order to create the same amount of recognizable SG stars, inhomogeneous mixing models require a FG mass 10-70% larger than in the homogeneous case (see the Appendix).

4 CONSTRAINTS ON THE TEMPORAL EVOLUTION OF DILUTION

In the previous section we considered the case of a globular cluster moving in the halo through the ambient medium that can be captured via the Bondi accretion (e.g. Lin & Murray 2007). In addition to this process, the moving cluster may accrete further material sweeping up the surrounding gas which impinges on a seed of AGB ejecta present in the GC centre. Both the above accretion mechanisms may work only for a limited range of values of the parameters of interest. In fact, if the energy of the incoming gas is much larger than the GC potential well, a collective accretion is prevented. Moreover, if the cluster velocity is too large, the ram pressure exerted on the gas at the bottom of the potential well becomes stronger than the gravitational restoring force, and the sweeping process clears out the cluster of its gas instead of promoting accretion (see Lin & Murray 2007).

A full hydrodynamic study of the processes depicted above will be presented in a separate paper (D'Ercole et al. in preparation). Here we want to constrain the accretion

[3] Note that we have minimized the dilution posing $M_{\rm SG}/M_{\rm FG} = 1$; assuming a larger ratio $M_{\rm SG}/M_{\rm FG}$ (as usually observed) would lead to an even more irrelevant pollution.

[4] Actually, the separating value is not the same for all the clusters, but is 0.4 dex larger than the minimum value of [Na/Fe] observed in the cluster.

[5] In models in which the FG is at least ten times more massive than the SG, the above condition becomes $\zeta \lesssim 0.1(1+\xi)/\xi$, and is easily met for any value of ξ.

[6] in order to maximize the effect of the inhomogeneity, we assumed $\alpha = 1.5$, α being a parameter regulating the shape of N([Na/Fe]) (see the Appendix).

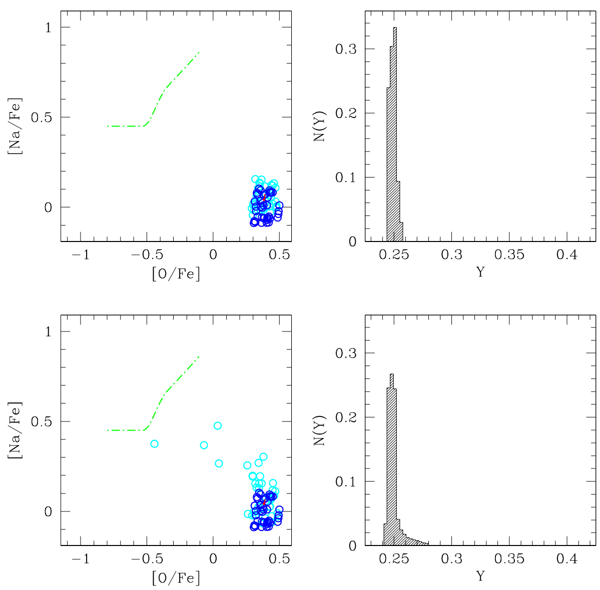

Figure 1. Model with uniform accretion with the following parameters:$(t_{\rm end,7}, \rho_{*,\rm FG}, f_{\rm pr}, \nu, x) = (10, 700, 1, 1, 0.5)$ Top-left panel: chemical evolution of the homogeneous model in the plane [O/Fe]-[Na/Fe]. In this panel the dot-dashed line represents the ISM chemical pattern in absence of any accretion of pristine gas, and the dashed line (barely visible) the actual GC evolution for the assumed dilution. The FG stars are represented by the blue circles, and the SG stars by the cyan circles. Top-right panel: helium distribution of the homogeneous model. Bottom-left and bottom-right panels are similar to the upper ones, but in case of inhomogeneous mixing.

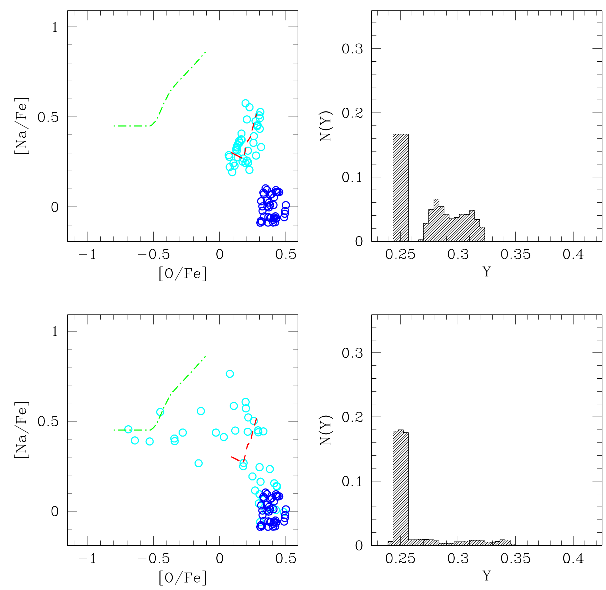

Figure 2. Model similar to that illustrated in figure 1, but with $f_{\rm pr} = 0.05$. Top panels: homogeneous case. Bottom panels: inhomogeneous case.

process on the basis of chemical arguments. If GCs actually accrete gas from the ambient medium continuously since the beginning, as suggested by Conroy & Spergel (2010), dilution starts quite soon and the formation of the Extreme population is inhibited because the stars of this population are extremely O-poor, and therefore are expected to be formed in the pure ejecta of the massive AGB stars.

In order to explicate this point, we run a model similar to that shown in the previous section, but assuming an amount of accreted matter which is only 5% of the FG mass, a characteristic value in the models shown in Paper I: we thus assume $(t_{\rm end,7}, \rho_{*,\rm FG}, f_{\rm pr}, \nu, x) = (10, 700, 0.05, 1, 0.5)$. As expected, in the homogeneous mixing case Fig. 2 (upper panel) shows that the stars have all [O/Fe]>0. Once the inhomogeneous mixing is allowed (see Fig. 2, lower panel), some O-poor stars actually form, but they remain too few to fit the amount of the Extreme (and Intermediate) stars in the GC where they are observed, reaching 15-25% of the total (D'Antona & Caloi 2008). This is well illustrated by the helium distribution function which barely reaches $Y \sim 0.33$ in the model, while nearly 20% of the stars in NGC 2808 has $Y > 0.35$ (Carretta et al. 2009; D'Antona & Caloi 2008). Moreover, comparing the two distribution functions in Fig. 2, we note that in the inhomogeneous case the peak at $Y \sim 0.25$ given by the FG stars is *higher*, reducing the ratio $M_{\rm SG}/M_{\rm FG}$ with respect to the homogeneous case. This is an effect due to the spread of the SG population which partially overlaps the Primordial stars (see the discussion in the Appendix).

The above argument rules out not only continuous accretion of external pristine gas, but also the *internal* continuous dilution suggested by Gratton & Carretta (2010) with the pristine gas produced by mass lost from less evolved stars (typically - but not only - MS stars) than those producing the polluting material.

We thus conclude that the dilution can not be achieved by a continuous mixing of pristine gas, at least for those clusters showing the presence of an Extreme population. We remind here that the models shown in Paper I may form O-poor stars because the bulk of the accretion occurs at a time $t_{\rm ac}$ larger than the time at which the massive AGB stars start to pollute the cluster. Therefore, accretion seems to be a quite episodic event, rather than a regular one.

5 CONCLUSIONS

In this paper we highlighted some characteristics of the dilution process occurring in the CGs on the basis of their chemical properties. Our findings can be summarized as follows:

(i) In principle, one could reproduce the ubiquitous O-Na anticorrelation present in the GCs without any dilution, simply assuming that all the available AGB models are incorrect and that actually the ejecta of the smaller AGB stars are less abundant in sodium and more abundant in oxygen. However, we have shown that this option appears to be ruled out by the direct correlation between $[{\rm O/Fe}]_{\rm min}$-$[{\rm Na/Fe}]_{\rm max}$ found by Carretta et al. (2009). We thus conclude that dilution must have actually occurred in GCs.

(ii) We have studied the dependence of the extent of the Na-O anticorrelation on the amount of pristine gas involved in the SG formation. Our analysis shows that the contribution of pristine gas to the matter from which SG stars form can not be larger than $\sim 0.1 M_{\rm FG}$. This confirms one of the

conclusions of our previous papers (Paper I; D'Ercole et al. 2008; Vesperini et al. 2010): in order to form the amount of SG stars observed today, clusters must have been initially about ten times more massive. Models suggesting an initial mass similar to the present one, and based on a flat ISM (D'Antona & Caloi 2004; Decressin et al. 2007a), have been ruled out by D'Ercole et al. (2008). A model recently proposed by Conroy & Spergel (2010) suggests that clusters would not need to be initially much more massive than today since SG stars would form almost entirely from a large amount of accreted pristine gas. Following Renzini (2008), such a model can be easily ruled out for those massive clusters containing a very helium rich population, because a cluster with a FG mass initially similar to the present one can not release the needed helium mass. The results presented in this paper imply that the model of Conroy & Spergel (2010) is ruled out also for all the other clusters without any Extreme Helium-rich population. In fact, if the accreted mass is of the same order of the FG, the abundances of the SG stars are almost identical to those of the FG stars, thus preventing the formation of the O-Na anticorrelation which instead is present in all GCs. Even a possible inhomogeneous mixing between the AGB ejecta and the accreted pristine gas, as invoked by Conroy & Spergel (2010), can not give rise to the Extreme SG population and to a substantial Intermediate SG population.

(iii) We demonstrated that if the dilution starts at the same time of the pollution, the Extreme stars can not form, because such O-poor stars are presumably built in the pure ejecta of massive AGB stars. Thus, some proposed accretion processes such as the Bondi accretion and streaming (Conroy & Spergel 2010), or the self-dilution due to unevolved stars (Gratton & Carretta 2010), can not work, at least for clusters hosting an Extreme population. Instead, dilution should be considered as a delayed episode restricted in time, as described in Paper I.

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APPENDIX A: INHOMOGENEOUS MIXING

In order to account for a possible inhomogeneous mixing between the pristine gas and the AGB ejecta we adopt the following scheme. We assume that both the pristine gas and the ejecta are clumpy, and the number of these clumps or clouds is distributed in mass following a power law:

$$\frac{dM_{\rm pr}}{dM} = AM^{1-\alpha} \tag{A1}$$

$$\frac{dM_{\rm ej}}{dM} = BM^{1-\beta}, \tag{A2}$$

where $dM_{\rm pr}$ ($dM_{\rm ej}$) is the mass of pristine gas (ejecta) included in clouds with masses between M and $M + dM$. As a consequence, the total mass $M_{\rm pr,tot}$ of pristine gas, and $M_{\rm ej,tot}$ of the ejecta, are given by

$$M_{\rm pr,tot} = \frac{A}{2-\alpha}\left(M_{\rm pr,u}^{2-\alpha} - M_{\rm pr,l}^{2-\alpha}\right), \tag{A3}$$

$$M_{\rm ej,tot} = \frac{B}{2-\beta}\left(M_{\rm ej,u}^{2-\beta} - M_{\rm ej,l}^{2-\beta}\right) \tag{A4}$$

where $M_{\rm pr,u}$ ($M_{\rm ej,u}$) and $M_{\rm pr,l}$ ($M_{\rm ej,l}$)are the lower and upper extremes of the pristine gas (ejecta) distribution, and A and B are normalization constants. As in our program we form stars as a sequel of "bursts" occurring after every time interval Δt, $M_{\rm pr,tot}$ and $M_{\rm ej,tot}$ represent the amount of pristine gas and ejecta present during Δt, and are given by the simulation. As a consequence, taking $M_{\rm pr,u}$, $M_{\rm ej,u}$, $M_{\rm pr,l}$, $M_{\rm ej,l}$, α and β as free parameters, A and B are determined. Choosing an uniform deviate $m_{\rm pr}$ in the range $0 < m_{\rm pr} < M_{\rm pr,tot}$ we generate a random mass $M_{\rm pr}$ of pristine cloud from the distribution in eq. A1, and we proceed

similarly to obtain a random value of the ejecta cloud $M_{\rm ej}$ (e.g. Press et al. 1992):

$$M_{\rm pr} = \left(\frac{2-\alpha}{A} m_{\rm pr} + M_{\rm pr,l}^{2-\alpha}\right)^{1/(2-\alpha)} \tag{A5}$$

$$M_{\rm ej} = \left(\frac{2-\beta}{B} m_{\rm ej} + M_{\rm ej,l}^{2-\beta}\right)^{1/(2-\beta)} . \tag{A6}$$

Assuming that these two clouds merge together, the resulting abundance $\mathcal{Z}$ in this mixed gas is given by

$$\mathcal{Z} = \frac{\mathcal{Z}_{\rm pr} M_{\rm pr} + \mathcal{Z}_{\rm ej} M_{\rm ej}}{M_{\rm pr} + M_{\rm ej}}, \tag{A7}$$

where $\mathcal{Z}_{\rm pr}$ and $\mathcal{Z}_{\rm ej}$ represent the abundance of the element in the pristine gas and in the ejecta, respectively.

After every Δt we chose a number of couples $(M_{\rm pr}, M_{\rm ej})$ obtaining the $\mathcal{Z}$-distribution of the stars forming at that time.

As pointed out above, we have six free parameters that must be fixed. For illustration, we assume $M_{\rm pr,u} = 10^4 M_{\rm pr,l}$, $M_{\rm pr,l} = 1$ M$_\odot$. For the sake of simplicity, we further assume $\beta = \alpha$, $M_{\rm ej,u}/M_{\rm ej,l} = M_{\rm pr,u}/M_{\rm pr,l}$, and $N_{\rm pr,tot} = N_{\rm ej,tot}$ (i.e. the number of pristine clouds is equal to the clouds of the ejecta), thus obtaining $M_{\rm ej,l}/M_{\rm pr,l} = M_{\rm ej,tot}/M_{\rm pr,tot} = x$ which is given by the simulation at every Δt.

In order to show the behaviour of the distribution for different values of α, Fig. A1 displays several histograms of the number of stars N([Na/Fe]) as a function of [Na/Fe] forming in a mixture of two gases, one with [Na/Fe]=0,representing the pristine gas, and one with [Na/Fe]=0.82, representing the AGB ejecta. Two ratios between the masses of these two gases are taken into account: $x = 1$, characteristic of models like those in Paper I, and $x = 0.05$, typical of models as those suggested by Conroy & Spergel (2010), in which the original FG mass is the same observed today.

We note that very steep ($\alpha = 3$) and very flat ($\alpha = 0.1$) mass functions give rise to similar distributions. For low values of α the number of small clouds and massive clouds are similar, thus the probability of encounters between clouds with similar mass is elevated, and the final stellar distribution peaks around the mean value which would be obtained in case of homogeneous mixing (vertical solid lines in Fig. A1). In case of high values of α, the majority of the mass is distributed in small clouds of similar mass; merging between these clouds are the most probable, and once more the final distribution peaks at the mean value. For intermediate values of α, the larger clouds compensate their lower number with their larger mass. When they make the most probable merging with a small cloud, a large number of stars form, having a value of [Na/Fe] very skewed toward the value of the large cloud; for this reason N([Na/Fe]) shows a clear bimodal structure with two peaks close to the edges of the [Na/Fe] range of values.

The vertical dashed lines in Fig. A1 indicate the value [Na/Fe]=0.4 which is adopted by Carretta et al. (2010) to separate the FG from the SG (see section 3). The figure clearly shows that, for $x = 0.05$, inhomogeneous mixing can not populate the SG region ([Na/Fe]$>$ 0.4) for any choice of α. This conclusion does not change even if we relax some of the other assumptions described above (and which play a minor role in shaping N([Na/Fe]).

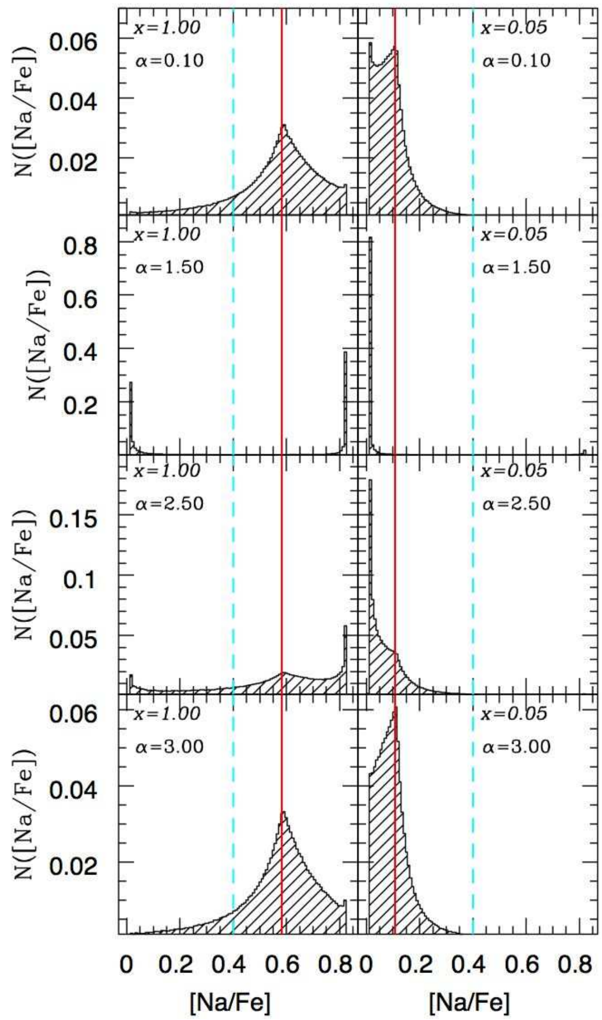

Figure A1. Different distributions of N([Na/Fe]) for different values ofα and x. The vertical solid lines indicate the mean value in case of homogeneous mixing (N([Na/Fe]) would be shaped as a delta dirac function). The dashed lines separate the FG population ([Na/Fe]$<$ 0.4) from the SG population ([Na/Fe]$>$ 0.4)(Carretta et al. 2010). For $x = 0.05$ the inhomogeneous mixing is notable to populate the SG region for any value of α.

We investigate now the influence of a possible inhomogeneous mixing models with $x \sim 1$, that is, in models in which the initial FG mass was much higher than today. Figure A2 illustrates the fraction ζ of SG stars with [Na/Fe]$>$ 0.4 present in N([Na/Fe]) as a function of x, for different values of α. It is clearly shown that, for large values of x, the fraction of new stars really appearing as SG stars to an observer oscillates in a range 0.6-0.9,depending on α.

From equation 1 (see text), the apparent ratio between the two populations is

$$0 < \frac{M_{\rm SG}}{M_{\rm FG}} < \xi \tag{A8}$$

depending on the value of ζ. Note that the maximum ratio is obtained for $\zeta = 1$, that is for the homogeneous mixing, when all the SG stars have [Na/Fe]$>$ 0.4 (see the vertical solid lines in the left panels of Fig. A1).

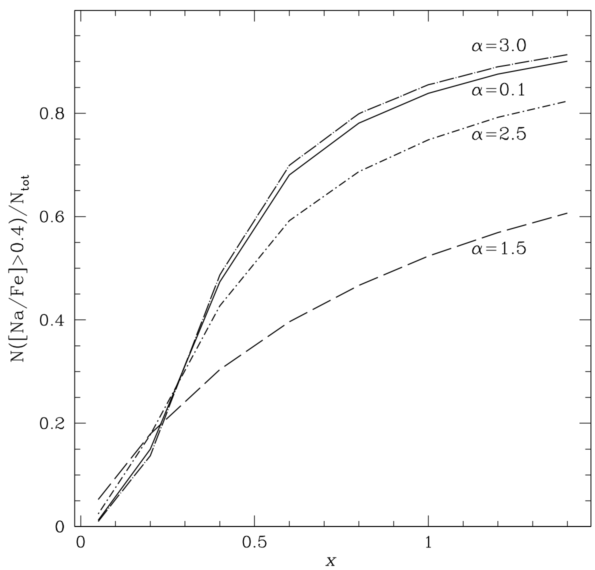

Figure A2. Fraction ζ of SG stars with [Na/Fe]>0.4 obtained by inhomogeneous mixing as function of x, and for different values of α.

Assuming $\xi = 0.05$ (see section 3), it turns out from equation 1 that the apparent ratio between the masses of the two populations is a fraction $\sim \zeta$ of the true value (holding in the homogeneous case). As a consequence, models with inhomogeneous mixing need the initial FG mass to be increased by a factor 1.1-1.7 with respect to homogeneous models to obtain the same amount of recognizable $M_{\rm SG}$.